ARIMA processes

PRACTICAL TIME SERIES ANALYSIS

THISTLETON AND SADIGOV

Objectives

- Describe autoregressive, integrated, moving average models
- Rewrite autoregressive, integrated, moving average models using backshift and difference operators

ARMA processes

Remember ARMA(p, q) process

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + Z_t + \beta_1 Z_{t-1} + \cdots \beta_q Z_{t-q}$$

can be written as

$$\phi(B) X_t = \beta(B) Z_t$$

where

$$\beta(B) = \beta_0 + \beta_1 B + \cdots + \beta_q B^q$$

$$\phi\,(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$$

- z – complex variable
- Roots of the polynomials $\beta(z)$ and $\phi(z)$ lie outside of the unit circle
- ARMA(p, q) process will be stationary and invertible

Non-stationary data

- Real life datasets are non stationary
- They might have a systematic change in trend
- We need to remove trend
- Difference operator $\nabla = 1 - B$

Difference operator

Remember

$$\nabla X_t = X_t - X_{t-1} = (1 - B)X_t$$

So, the random walk model

$$X_t = X_{t-1} + Z_t$$

can be written

$$\nabla X_t = Z_t$$

ARIMA(p, d, q) process

A process X_t is Autoregressive INTEGRATED Moving Average of order (p, d, q) if

$$Y_t := \nabla^d X_t = (1 - B)^d X_t$$

is ARMA(p, q).

$$Y_t \sim ARMA(p, q)$$

$$X_t \sim ARIMA(p, d, q)$$

$$\phi(B)\nabla^d X_t = \beta(B)Z_t$$

or

$$\phi(B)(1-B)^d X_t = \beta(B)Z_t$$

d – order of differencing

- $d = 1$ or $d = 2$
- Over differencing may introduce dependence
- ACF might also suggest differencing is needed
- $\phi(z)(1 - z)^d$ has unit root with multiplicity of d
- ACF will decay very slowly

Modeling

- Trend suggests differencing
- Variation in variance suggests transformation
- Common transformation: log, then differencing
- It is also known as log-return
- ACF suggests order of moving average process (q)
- PACF suggests order of autoregressive process (p)
- Akaike Information Criterion (AIC)
- Sum of squared errors (SSE)
- Ljung-Box Q-statistics (Next lecture)
- Estimation!

What We've Learned

- Describe autoregressive, integrated, moving average models
- Rewrite autoregressive, integrated, moving average models using backshift and difference operators