Ljung-Box Q-statistic

PRACTICAL TIME SERIES ANALYSIS

THISTLETON AND SADIGOV

Objectives

- Define Ljung-Box Q-statistic
- Learn the decision rule to test the null hypothesis that several autocorrelation coefficients are zero
- Test the null hypothesis that several autocorrelation coefficients are zero using R

Portmanteau statistic

Box and Pierce (1970) proposed Portmanteau statistic

$$Q^*(m) = T\sum_{l=1}^{m} r_l^2$$

as a test statistic for the null hypothesis

$$H_0: \rho_1 = \rho_2 = \cdots = \rho_m = 0$$

against the alternative hypothesis

$$H_a: \rho_i \neq 0$$

for some $i \in \{1,2, \ldots, m\}$.

Under i.i.d condition of $\{r_t\}$,

$$Q^*(m) \sim \chi^2(df = m)$$

asymptotically.

Ljung and Box (1978) modified statistic to increase the power of the test in finite samples

$$Q(m) = T(T+2)\sum_{l=1}^{m}\frac{r_l^2}{T-l}$$

Decision rule

We reject the null hypothesis if $\mathrm{Q}(m)$ is large enough i.e.,

$$Q(m) > \chi^2_\alpha$$

where χ^2_α is $100(1-\alpha)$-th quantile of Chi-Squared distribution with m degrees of freedom.

Most packages will actually calculate p-value. We will reject the null hypothesis if the p-value is sufficiently small, i.e.

$$p < \alpha$$

where α is the significance level.

Choice of m and R routine

Usually we take

$$m \approx \ln(T)$$

R routine

$$Box.test(data, lag = \log(T))$$

What We've Learned

- Define Ljung-Box Q-statistic
- Learn the decision rule to test the null hypothesis that several autocorrelation coefficients are zero
- Test the null hypothesis that several autocorrelation coefficients are zero using R