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Advanced Kalman Filtering and Sensor Fusion

Extended Kalman Filter Summary

System Model:

$$
\begin{aligned}
x_k &= \mathbf{f}(x_{k-1}, u_k, w_k) \\
z_k &= \mathbf{h}(x_k, v_k)
\end{aligned}
$$

Assumptions:

$$
\underbrace{\begin{aligned} w_k &\sim N(0, \mathbf{Q}_k) \\ v_k &\sim N(0, \mathbf{R}_k) \end{aligned}}
$$

Gaussian Distribution with **zero mean** and given **covariance matrix**

$$
\left.\begin{aligned} E(w_k w_j^T) &= \mathbf{Q}_k \delta_{k-j} \\ E(v_k v_j^T) &= \mathbf{R}_k \delta_{k-j} \end{aligned}\right\} \text{Not correlated with time}
$$

$$
E(w_k v_j^T) = 0
$$

Process noise and Measurement noise are independant

Predict:

$$
\begin{aligned}
\hat{x}_k^- &= f(\hat{x}_{k-1}^+, u_k, 0) \\
\mathbf{P}_k^- &= \nabla f_x \mathbf{P}_{k-1}^+ \nabla f_x{}^T + \nabla f_w \mathbf{Q}_k \nabla f_w{}^T
\end{aligned}
$$

Update:

$$
\begin{aligned}
\nu_k &= z_k - h(\hat{x}_k^-, 0) \\
S_k &= \nabla h_x P_k^- \nabla h_x^T + \nabla h_v R_k \nabla h_v^T \\
\mathbf{K}_k &= \mathbf{P}_k^- \nabla h_x^T \mathbf{S}_k^{-1} \\
\hat{x}_k^+ &= \hat{x}_k^- + \mathbf{K}_k \nu_k \\
\mathbf{P}_k^+ &= (\mathbf{I} - \mathbf{K}_k \nabla h_x) \mathbf{P}_k^-
\end{aligned}
$$

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Process Model:

$$x_k = \mathbf{f}(x_{k-1}, u_k, w_k)$$

$$\nabla f_x = \frac{\partial f}{\partial x}\bigg|_{x=\hat{x}_{k-1}^{+}}$$

$$\nabla f_w = \frac{\partial f}{\partial w}\bigg|_{x=\hat{x}_{k-1}^{+}}$$

Measurement Model:

$$z_k = \mathbf{h}(x_k, v_k)$$

$$\nabla h_x = \frac{\partial h}{\partial x}\bigg|_{x=\hat{x}_{k}^{-}}$$

$$\nabla h_v = \frac{\partial h}{\partial v}\bigg|_{x=\hat{x}_{k}^{-}}$$

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Extended Kalman Filter Summary

Discrete - Time Extended Kalman Filter Algorithm

1. The system and measurement equations are given in the following form:

$$\begin{aligned} x_k &= \mathbf{f}(x_{k-1}, u_k, w_k) \\ z_k &= \mathbf{h}(x_k, v_k) \\ w_k &\sim N(0, \mathbf{Q}_k) \\ v_k &\sim N(0, \mathbf{R}_k) \end{aligned}$$

2. The filter is initialised as follows:

$$\begin{aligned} \hat{x}_0^+ &= E[x_0] \\ P_0^+ &= E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \end{aligned}$$

3. For each time step k = 1,2, …, perform the follow prediction/update steps:

   a. Calculate the process model Jacobian Matrices for time k - 1 using:

$$\begin{aligned} \nabla f_x &= \left.\frac{\partial f}{\partial x}\right|_{x=\hat{x}_{k-1}^+} \\ \nabla f_w &= \left.\frac{\partial f}{\partial w}\right|_{x=\hat{x}_{k-1}^+} \end{aligned}$$

   b. Perform the Kalman Filter state and covariance prediction step calculations:

$$\begin{aligned} \hat{x}_k^- &= f(\hat{x}_{k-1}^+, u_k, 0) \\ \mathbf{P}_k^- &= \nabla f_x \mathbf{P}_{k-1}^+ \nabla f_x{}^T + \nabla f_w \mathbf{Q}_k \nabla f_w{}^T \end{aligned}$$

Extended Kalman Filter Summary

Discrete - Time Extended Kalman Filter Algorithm (Cont)

c. Calculate the measurement model Jacobian matrices for time k using:

$$\nabla h_x = \frac{\partial h}{\partial x}\Big|_{x=\hat{x}_k^-}$$
$$\nabla h_v = \frac{\partial h}{\partial v}\Big|_{x=\hat{x}_k^-}$$

d. Perform the Kalman Filter state and covariance measurement update step calculations:

$$\nu_k = z_k - h(\hat{x}_k^-, 0)$$
$$S_k = \nabla h_x P_k^- \nabla h_x^T + \nabla h_v R_k \nabla h_v^T$$
$$\mathbf{K}_k = \mathbf{P}_k^- \nabla h_x^T \mathbf{S}_k^{-1}$$
$$\hat{x}_k^+ = \hat{x}_k^- + \mathbf{K}_k \nu_k$$
$$\mathbf{P}_k^+ = (\mathbf{I} - \mathbf{K}_k \nabla h_x)\,\mathbf{P}_k^-$$